

Topological Black Holes of Einstein-Yang-Mills dilaton Gravity

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Abstract

We present the topological solutions of Einstein-dilaton gravity in the presence of a non-Abelian Yang-Mills field. In 4 dimensions, we consider the $So(3)$ and $So(2, 1)$ semisimple group as the Yang-Mills gauge group, and introduce the black hole solutions with spherical and hyperbolic horizons, respectively. The solution in the absence of dilaton potential is asymptotically flat and exists only with spherical horizon. Contrary to the non-extreme Reissner-Nordstrom black hole, which has two horizons with a timelike and avoidable singularity, here the solution may present a black hole with a null and unavoidable singularity with only one horizon. In the presence of dilaton potential, the asymptotic behavior of the solutions is neither flat nor anti-de Sitter. These solutions contain a null and avoidable singularity, and may present a black hole with two horizons, an extreme black hole or a naked singularity. We also calculate the mass of the solutions through the use of a modified version of Brown and York formalism, and consider the first law of thermodynamics.

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I. INTRODUCTION

In the low energy limit of the string theory, one recovers Einstein gravity along with a scalar dilaton field which is non-minimally coupled to a matter field [1]. A typical feature of the dilaton in string theory is its exponential coupling to the matter field in the frame where the dilaton decouples from the Ricci scalar. Exact solutions for charged dilaton black holes in which the dilaton is coupled to the Maxwell field have been constructed by many authors. It is found that the presence of dilaton has important consequences on the asymptotic behavior and the thermodynamic properties of the black hole solutions. The asymptotically flat solutions of Einstein-Maxwell-dilaton (EMD) theory have been investigated in [2]. Solutions of EMD theory with one Liouville-type potential which are neither asymptotically flat nor anti-de Sitter (AdS) have been considered in [3]. These kinds of solutions with three Liouville-type potentials have been considered in [4].

Although the gauged supergravity AdS theories generically contain Yang-Mills (YM) fields, most of the studies in the literature have been restricted to the case of Abelian gauge fields in the bulk. In this work we consider Einstein-dilaton gravity in the presence of the non-Abelian Yang-Mills field and investigate the existence of exact solutions with different asymptotic behavior. In the absence of dilaton, this theory with $SU(2)$ gauge group has both solitonic [5] and colored black hole solutions [6], while Einstein-Yang-Mills (EYM) with $So(N)$ or $So(N - 1, 1)$ gauge groups has only black hole solutions [7–9]. These solutions have led to certain revisions of some of the basic concepts of black hole physics based on the uniqueness and no-hair theorem. It is now well-known that this theory possesses "hairy" black hole solutions, whose metric is not a member of the Kerr-Newmann family (see [10] for a detailed review in 4 dimensions and [11] for a recent review in higher dimensions). Solutions of the EYM equations were also investigated in the presence of cosmological constant [12, 13]. In the presence of dilaton, the solutions of Einstein-Yang-Mills-dilaton (EYMD) theory have been considered by many authors [14]. The presence of a dilaton field also invites the topological black holes into the game. Indeed, the horizon topology of an asymptotically flat black hole should be a round sphere, while in the presence of dilaton with a potential, it is possible to have a black holes with zero or negative constant curvature horizon too. These black holes are referred to as topological black holes in the literature. These kinds of solutions in EMD theory have been investigated in [15]. Here, we present the black hole

solutions of Einstein-dilaton gravity in the presence of non-Abelian Yang-Mills field with $SO(3)$ and $SO(2,1)$ gauge group with different asymptotic behavior. We use the Wu-Yang ansatz [16] in which the Yang-Mills (YM) gauge potential have only angular components.

The outline of the work is as follows. In Sec. II, we give a brief review of the field equations of Einstein-dilaton gravity in the presence of YM gauge fields with a semisimple gauge group. In Sec. III we present static solutions with spherical and hyperbolic horizons for gauge fields with $So(3)$ and $So(2,1)$ gauge group. We calculate the mass of the solutions through the use of a modified Brown and York formalism and investigate the first law of thermodynamics. Finally, we give some concluding remarks.

II. FIELD EQUATIONS

Here, we review the field equations of Einstein gravity coupled with dilaton and non-Abelian Yang-Mills field with an N -parameters gauge group \mathcal{G} , which is assumed to be at least semisimple with structure constants C_{bc}^a and metric tensor $\Gamma_{ab} = C_{ad}^c C_{bc}^d$. The action of this theory may be written as

$$I_G = -\frac{1}{16\pi} \int_{\mathcal{M}} d^{n+1}x \sqrt{-g} \left(\mathcal{R} - \frac{4}{n-1} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi) - e^{-4\alpha\Phi/(n-1)} \gamma_{ab} F_{\mu\nu}^{(a)} F^{(b)\mu\nu} \right) - \frac{1}{8\pi} \int_{\partial\mathcal{M}} d^n x \sqrt{-h} K, \quad (1)$$

where the Latin indices a, b, \dots go from 1 to N , and the repeated indices is understood to be summed over, \mathcal{R} is the Ricci scalar curvature, Φ is the dilaton field, $V(\Phi)$ is a potential for Φ and

$$\gamma_{ab} \equiv -\frac{\Gamma_{ab}}{|\det \Gamma_{ab}|^{1/N}}. \quad (2)$$

In Eq. (1) $F_{\mu\nu}^{(a)}$'s are the non-Abelian gauge fields:

$$F_{\mu\nu}^{(a)} = \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)} + \frac{1}{2e} C_{bc}^a A_\mu^{(b)} A_\nu^{(c)}, \quad (3)$$

where e is a coupling constant and $A_\mu^{(a)}$'s are the gauge potentials. The last term in Eq. (1) is the Gibbons-Hawking boundary term which is chosen such that the variational principle is well-defined, and K is the trace of the extrinsic curvature K^{ab} of any boundary(ies) $\partial\mathcal{M}$ of the manifold \mathcal{M} , with induced metric(s) h_{ab} . Variation of the action (1) with respect to the gauge potential $A_\mu^{(a)}$, the spacetime metric $g_{\mu\nu}$ and the dilaton field Φ yield the EYMD

equations as

$$F^{(a)\mu\nu}{}_{;\nu} = \frac{4\alpha}{n-1} F^{(a)\mu\nu} \Phi_{;\nu} + \frac{1}{e} C_{bc}^a A_\nu^{(b)} F^{(c)\nu\mu}, \quad (4)$$

$$\begin{aligned} \mathcal{R}_{\mu\nu} = & \frac{4}{n-1} \left(\partial_\mu \Phi \partial_\nu \Phi + \frac{1}{4} g_{\mu\nu} V(\Phi) \right) \\ & + 2 \exp \left(-\frac{4\alpha\Phi}{n-1} \right) \gamma_{ab} \left(F_\mu^{(a)\lambda} F_{\nu\lambda}^{(b)} - \frac{1}{2(n-1)} g_{\mu\nu} F^{(a)\lambda\sigma} F_{\lambda\sigma}^{(b)} \right), \end{aligned} \quad (5)$$

$$\nabla^2 \Phi = \frac{n-1}{8} \frac{\partial V}{\partial \Phi} - \frac{\alpha}{2} \exp \left(-\frac{4\alpha\Phi}{n-1} \right) \gamma_{ab} F_{\lambda\eta}^{(a)} F^{(b)\lambda\eta}, \quad (6)$$

III. STATIC BLACK HOLES

The 4-dimensional metric of a spherically symmetric spacetime may be written as

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + R^2(r)d\Omega^2, \quad (7)$$

where

$$\begin{aligned} d\Omega^2 &= d\theta^2 + \sin^2 \theta d\varphi^2; & k &= 1, \\ &= d\theta^2 + \sinh^2 \theta d\varphi^2; & k &= -1, \end{aligned} \quad (8)$$

for spherical and hyperbolic horizons, respectively. Here, we consider the static case, and therefore we assume that Φ depends on r only. For the spacetime with spherical horizon ($k = 1$), the $So(3)$ YM potentials

$$\begin{aligned} A_\mu^{(1)} &= e (-\cos \varphi d\theta + \sin \theta \cos \theta \sin \varphi d\varphi), \\ A_\mu^{(2)} &= -e (\sin \varphi d\theta + \sin \theta \cos \theta \cos \varphi d\varphi), \\ A_\mu^{(3)} &= e \sin^2 \theta d\varphi, \end{aligned} \quad (9)$$

with non-zero components of YM field tensor as

$$\begin{aligned} F^{(1)\theta\varphi} &= -\frac{e \sin \varphi}{R^4(r)}, \\ F^{(2)\theta\varphi} &= \frac{e \cos \varphi}{R^4(r)}, \\ F^{(3)\theta\varphi} &= \frac{e \cot \theta}{R^4(r)}. \end{aligned} \quad (10)$$

satisfy the YM equation (4). For the spacetime with hyperbolic horizon, $k = -1$, the $So(2, 1)$ gauge fields

$$\begin{aligned} A_\mu^{(1)} &= e(-\cos\varphi d\theta + \sinh\theta \cosh\theta \sin\varphi d\varphi), \\ A_\mu^{(2)} &= -e(\sin\varphi d\theta + \sinh\theta \cosh\theta \cos\varphi d\varphi), \\ A_\mu^{(3)} &= e\sinh^2\theta d\varphi, \end{aligned} \quad (11)$$

satisfy the YM field equation (4).

In order to solve the field equation (5) for three unknown functions $f(r)$, $R(r)$ and $\Phi(r)$, we make the ansatz

$$R(r) = r \exp(-\alpha\Phi). \quad (12)$$

Using the above ansatz (12), the YM fields (9) or (11) and the metric (7), one can easily show that the components of Eq. (5) reduce to

$$(r^2 f')' - 2\alpha r^2 \Phi' f' + Vr^2 - 2e^2 r^{-2} \exp(2\alpha\Phi) = 0, \quad (13)$$

$$(r^2 f')' - 2\alpha r^2 \Phi' f' + Vr^2 - 2e^2 r^{-2} \exp(2\alpha\Phi) = 4rf \left[\alpha r \Phi'' - 2\alpha \Phi' + (1 + \alpha^2) r \Phi'^2 \right] \quad (14)$$

$$2(rf)' - 2\alpha r^2 (f\Phi')' + Vr^2 + 2(e^2 r^{-2} - k) \exp(2\alpha\Phi) + 4\alpha r f \Phi' (\alpha r \Phi' - 2) = 0, \quad (15)$$

where prime denotes the derivative with respect to r . Subtracting Eq. (13) from Eq. (14) gives

$$\alpha r \Phi'' - (1 + \alpha^2) r \Phi'^2 + 2\alpha \Phi' = 0, \quad (16)$$

with the solution

$$\Phi(r) = -\frac{\alpha}{(1 + \alpha^2)} \ln\left(1 - \frac{r_0}{r}\right), \quad (17)$$

where r_0 is an arbitrary constant. Thus, the metric (7) may be written as

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 \left(1 - \frac{r_0}{r}\right)^{2\alpha^2/(1+\alpha^2)} d\Omega^2, \quad (18)$$

which is physical only for $r \geq r_0$. Thus, one should restrict the spacetime to the region $r \geq r_0$, by introducing a new radial coordinate ρ as:

$$\rho^2 = r^2 - r_0^2 \Rightarrow dr^2 = \frac{\rho^2}{\rho^2 + r_0^2} d\rho^2 \quad (19)$$

With this new coordinate, the above metric becomes:

$$ds^2 = -f(\rho)dt^2 + \frac{\rho^2 d\rho^2}{(\rho^2 + r_0^2)f(\rho)} + (\rho^2 + r_0^2) \left(1 - \frac{r_0}{\sqrt{\rho^2 + r_0^2}}\right)^{2\alpha^2/(1+\alpha^2)} d\Omega^2, \quad (20)$$

which is now physical for $0 \leq \rho < \infty$. In the rest of the paper, we work in the r -coordinate for simplicity.

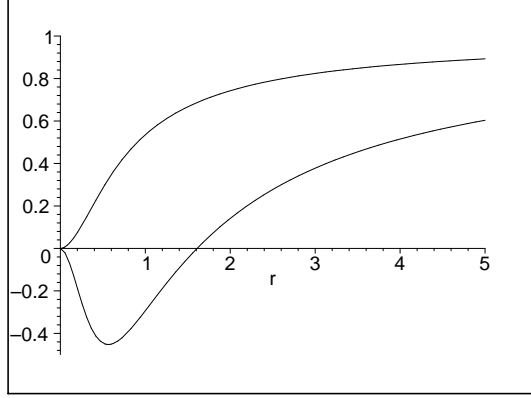


FIG. 1: $f(r)$ versus r for $\alpha = .2$, $r_0 = .5$, $e \leq e_{\text{ext}}$ and $e > e_{\text{ext}}$ from up to down, respectively.

A. Asymptotically flat solutions:

First, we consider the solutions with $V(\Phi) = 0$. For this case, the solution exists only for $k = 1$ with spherical horizon. Substituting $\Phi(r)$ in the field equations (13)-(15) one finds the function $f(r)$ as

$$f(r) = \left(1 - \frac{(1 + \alpha^2)e^2}{r_0 r}\right) \left(1 - \frac{r_0}{r}\right)^{(1-\alpha^2)/(1+\alpha^2)}. \quad (21)$$

One can show that the Kretschmann scalar $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ diverges at $\rho = 0$ ($r = r_0$), and therefore there is a curvature singularity located at $r = r_0$. The solution presents a black hole with horizon radius $r_+ = (1 + \alpha^2)e^2/r_0$, provided $e^2 > r_0^2/(1 + \alpha^2)$, and a naked singularity otherwise (see Fig. 1). Contrary to the Reissner-Nordstrom solution, which is the $\alpha = 0$ limit of this solution with two horizons, the solution given by Eqs. (18) and (21) has only one horizon. While the singularity of Reissner-Nordstrom is timelike and avoidable, here the singularity is null and unavoidable. This can be seen in the Penrose diagram of the solution, which is drawn in Fig. 2. The Hawking temperature of the black holes can be easily obtained by requiring the absence of conical singularity at the horizon in the Euclidean sector of the black hole solutions. One obtains

$$T_+ = \frac{1}{4\pi r_+} \left(1 - \frac{r_0}{r_+}\right)^{(1-\alpha^2)/(1+\alpha^2)}. \quad (22)$$

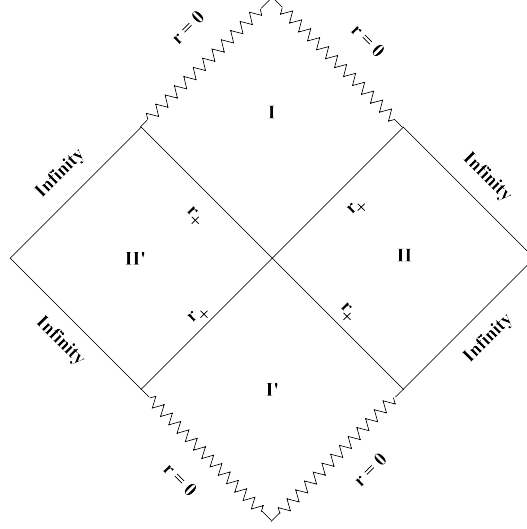


FIG. 2: Penrose diagram for $e > e_{\text{ext}}$.

B. Asymptotically non-flat solutions:

Here, we consider the solutions of EYMD theory in the presence of the following potential [4]

$$V(\Phi) = -\frac{1}{(1+\alpha^2)l^2} \left\{ -2\alpha^2(1-3\alpha^2)\exp\left(-\frac{2\Phi}{\alpha}\right) + 2(3-\alpha^2)\exp(2\alpha\Phi) + 16\alpha^2\exp\left(-\frac{\Phi(1+\alpha^2)}{\alpha}\right) \right\}. \quad (23)$$

Such a potential may arise from the compactification of a higher dimensional supergravity model which originates from the low energy limit of a background string theory [17]. With this potential, the solution of Eqs. (13)-(15) may be written as

$$f(r) = \left(k - \frac{(1+\alpha^2)e^2}{r_0 r}\right) \left(1 - \frac{r_0}{r}\right)^{(1-\alpha^2)/(1+\alpha^2)} + \frac{r^2}{l^2} \left(1 - \frac{r_0}{r}\right)^{2\alpha^2/(1+\alpha^2)}. \quad (24)$$

This metric at large r may be written as:

$$ds^2 = -f_0(r)dt^2 + \frac{dr^2}{f_0(r)} + r^2 d\Omega^2, \quad (25)$$

where

$$f_0(r) = k - \frac{\alpha^2 r_0^2}{l^2(1+\alpha^2)^2} + \left(\frac{r}{l} - \frac{\alpha^2}{1+\alpha^2} \frac{r_0}{l}\right)^2. \quad (26)$$

Thus, the asymptotic behavior of the spacetime in the presence of the potential (23) is not exactly AdS. Indeed the metric given by Eqs. (25) and (26) does not satisfy the Einstein

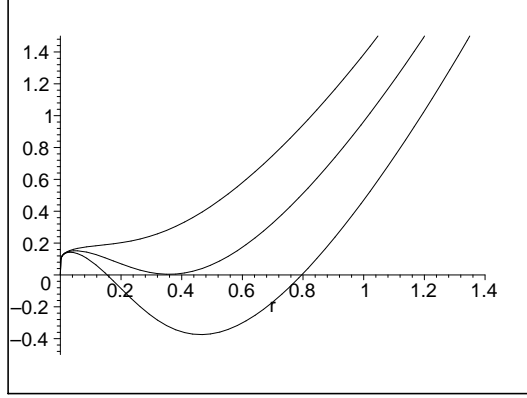


FIG. 3: $f(r)$ versus r for $\alpha = .2$, $r_0 = .5$, $e < e_{\text{ext}}$, $e = e_{\text{ext}}$ and $e > e_{\text{ext}}$ from up to down, respectively.

field equation in the presence of the cosmological constant. Again the solution given by Eqs. (18) and (24) has a curvature singularity at $r = r_0$, and presents a black hole with two horizons if $e > e_{\text{ext}}$, an extreme black hole when $e = e_{\text{ext}}$, and a naked singularity otherwise, where

$$e_{\text{ext}}^2 = \frac{r_0 r_{\text{ext}}}{1 + \alpha^2} \left\{ k + \frac{r_{\text{ext}}^2}{l^2} \left(1 - \frac{r_0}{r_{\text{ext}}} \right)^{(3\alpha^2 - 1)/(1 + \alpha^2)} \right\}. \quad (27)$$

In Eq. (27) r_{ext} is the root of $T_+ = 0$, where T_+ is the temperature of the black hole give by

$$T_+ = \frac{1}{4\pi r_+} \left(1 - \frac{r_0}{r_+} \right)^{(1 - \alpha^2)/(1 + \alpha^2)} \left\{ k + \frac{3(1 + \alpha^2)r_+^2 - 4br_+}{(1 + \alpha^2)l^2} \left(1 - \frac{r_0}{r_+} \right)^{-2(1 - \alpha^2)/(1 + \alpha^2)} \right\}. \quad (28)$$

Figure 3 shows the metric function as a function of r for various values of e . Here, again the singularity is null, but it is avoidable.

IV. MASS OF THE BLACK HOLES

It is well known that the gravitational action given in Eq. (1) diverges. A systematic method of dealing with this divergence for asymptotically AdS solutions is through the use of the counterterm method inspired by AdS/CFT correspondence. However, our solutions (21) or (24) are not asymptotically AdS. Thus, we use the subtraction method of Brown and York (BY) [18]. In order to use the BY method the metric should have the form

$$ds^2 = -W(R)dt^2 + \frac{dR^2}{V(R)} + R^2 d\Omega^2. \quad (29)$$

Thus, we should first write the metric (18) in the above form. To do this, we perform the following transformation:

$$R = r \left(1 - \frac{r_0}{r} \right)^{\alpha^2/(1+\alpha^2)}.$$

It is a matter of calculations to show that the metric (18) gets the same form as (29) with the following metric functions:

$$W(R) = f(r(R)), \quad (30)$$

$$V(R) = f(r(R)) \left(\frac{dR}{dr} \right)^2 = \left(1 - \frac{r_0}{(1+\alpha^2)r} \right) \left(1 - \frac{r_0}{r} \right)^{-1/(1+\alpha^2)} f(r(R)). \quad (31)$$

The background metric is chosen to be the metric (29) with:

$$W_0(R) = k - \frac{\alpha^2 r_0^2}{l^2(1+\alpha^2)^2} + \left(\frac{R}{l} - \frac{\alpha^2}{1+\alpha^2} \frac{r_0}{l} \right)^2, \quad (32)$$

$$V_0(R) = k + \left(\frac{R}{l} - \frac{\alpha^2}{1+\alpha^2} \frac{r_0}{l} \right)^2. \quad (33)$$

To compute the conserved mass of the spacetime, we choose a timelike Killing vector field ξ on the boundary surface \mathcal{B} with the metric (8). Then the quasilocal conserved mass can be written as

$$\mathcal{M} = \frac{1}{8\pi} \int_{\mathcal{B}} d^2\varphi \sqrt{\sigma} \{ (K_{ab} - K h_{ab}) - (K_{ab}^0 - K^0 h_{ab}^0) \} n^a \xi^b, \quad (34)$$

where σ is the determinant of the metric (8), K_{ab}^0 is the extrinsic curvature of the background metric and n^a is the timelike unit normal vector to the boundary \mathcal{B} . In the context of counterterm method, the limit in which the boundary \mathcal{B} becomes infinite (\mathcal{B}_∞) is taken, and the counterterm prescription ensures that the action and conserved charges are finite. Although the function $r(\mathcal{R})$ cannot be obtained explicitly, but at large R this can be done. Thus, one can calculate the mass through the use of the above modified Brown and York formalism as

$$M = \frac{(1+\alpha^2)^2 e^2 + (1-\alpha^2) r_0^2}{2(1+\alpha^2) r_0}. \quad (35)$$

In order to check the correctness of the above mass, we consider the first law of thermodynamics. The entropy of the dilaton black hole typically satisfies the so called area law of the entropy [19]. This near universal law which is applied to almost all kinds of black holes, including dilaton black holes, in Einstein gravity [20] gives

$$S = \pi r_+^2 \left(1 - \frac{r_0}{r_+} \right)^{2\alpha^2/(1+\alpha^2)}. \quad (36)$$

The parameter r_0 may be written in term of S and r_+ as

$$r_0 = r_+ \left\{ 1 - \left(\frac{S}{\pi r_+^2} \right)^{(1+\alpha^2)/2\alpha^2} \right\}. \quad (37)$$

Now using Eq. (37) and the fact that $f(r_+) = 0$, one may find the relation between S and r_+ as:

$$\left[1 - \left(\frac{S}{\pi r_+^2} \right)^{(1+\alpha^2)/2\alpha^2} \right] \left[k - \frac{r_+^2}{l^2} \left(\frac{S}{\pi r_+^2} \right)^{(3\alpha^2-1)/2\alpha^2} \right] - \frac{(1+\alpha^2)e^2}{r_+^2} = 0. \quad (38)$$

Since r_0 depends on S and r_+ , the mass M given in Eq. (35) depends on e , S and r_+ . But, S and r_+ are related by Eq. (38), and therefore M may be regarded as a function of the extensive parameters S and e . Using the fact that $dM(S, e) = (\partial M / \partial S)_e dS + (\partial M / \partial e)_S de$ one may show that

$$dM = T_+ dS + U de, \quad (39)$$

where T_+ is given in Eq. (28) and $U = e/r_+$. Thus, the mass calculated through the modified version of Brown and York formalism satisfies the first law of thermodynamics. It is worth to mention that the coupling constant e is the same as the charge Q for the solution of Einstein Maxwell gravity.

V. CLOSING REMARKS

We introduced the black hole solutions of Einstein-Yang-Mills-dilaton theory with spherical and hyperbolic horizons and investigate their properties. First, we presented an asymptotically flat solution in the absence of dilaton potential, for which the horizon should be a 2-sphere. It is worth to compare the distinguishing features of this solution with the asymptotically flat Reissner-Nordstrom solution. The singularity of asymptotically flat solution of Einstein-Maxwell theory is timelike and avoidable, while the singularity of the asymptotically flat solution presented here is null and unavoidable. This is shown in the Penrose diagram of the solution. Also, we found that one cannot have an extreme asymptotically flat black hole, while the extreme Reissner-Nordstrom black hole exists. Second, we investigated the asymptotically non-flat solutions in the presence of a dilaton potential. In this case, the horizon can also be a hyperbola, which is known as the topological solution. The asymptotic behavior of these solutions with spherical and hyperbolic horizons are neither flat nor AdS. The singularities of these solutions are null, but they are timelike and therefore avoidable.

These solutions may present a naked singularity, an extreme black hole or a black hole with two horizons.

As we mentioned, the solutions in the presence of dilaton potential are neither asymptotically flat nor AdS and therefore one cannot use the Brown and York formalism or AdS/CFT counterterm method to compute the mass of the black holes. To compute the mass of the solutions, we introduced a modified version of the Brown and York formalism and calculated the mass. We checked the correctness of the mass computed through the use of the modified version of Brown and York by investigating the first law of thermodynamics. We found that the calculated mass satisfies the first law of thermodynamics. In this paper, we only introduced the 4-dimensional solutions of Einstein-Yang-Mills theory in the presence of dilaton. Since higher-dimensional black holes have attracted a lot of interest, it is worth to introduce the higher-dimensional solutions in EYMD theory and investigate their properties.

Acknowledgements

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